## MULTIPLE CHOICE SOLUTIONS--MECHANICS

TEST III

1.) An atomic particle whose mass is 210 atomic mass units collides with a stationary particle whose mass is 40 atomic mass units. Which of the sketches below most closely approximates a possible collision scenario?
a.)

b.)

c.)

d.)

[Commentary: Due to its size, $A$ isn't going to deviate much upon colliding with $B$ (this effectively eliminates graphs $a$ and $b$ ). Due to momentum considerations, neither of the masses would be expected to move backwards after the collision (this eliminates graph d). Graph c comes closest.]
2.) A satellite orbits with a constant velocity vin a circular path around a planet. At one point in the flight, a short thruster burst is used to do work on the satellite, motivating it into an elliptical path. After the burst, the average velocity per orbit of the satellite is less than v. In what direction were the thrusters fired?
a.) In the direction of the satellite's motion. [Firing the thrusters forward in the direction of the satellite's motion will momentarily slow the satellite, causing it to move into an oblique elliptical path. As a consequence, the satellite will be pulled closer to the planet as it passed by and, as a consequence, will travel faster than otherwise expected at that point. In fact, the velocity throughout the motion will, on average, be faster than would have otherwise been the case. In short, this option is not true.]
b.) In the direction opposite the satellite's motion. [Firing the thrusters backward in the direction opposite the satellite's motion will momentarily speed the satellite up, causing it to move out farther away from the planet than would otherwise be expected. As a consequence, the satellite will pass farther from the planet at its closest point, moving more slowly in the process. In fact, the average orbital velocity will be slower than would have otherwise been the case. In short, this option will do the job.]
c.) Perpendicular to and inward relative to the satellite's motion. [I nteresting observation: a force applied perpendicular to the motion will do no work. According to the problem's statement, work was done. This option will not do.]
d.) There is not enough information given to tell. [Nope.]
3.) A jackhammer pounds the ground with a frequency of 5 hertz. If each cycle is driven by a hydraulic spring whose effective spring constant is $800 \mathrm{nt} / \mathrm{m}$, and if the throw of the device (i.e., the distance over which the bit is accelerated before contact) is .32 meters:
a.) The energy provided by each stroke will be approximately 40 joules. [The potential energy wrapped up in what is effectively a spring system here is . $5 \mathrm{kA}{ }^{2}$, where $k$ is the spring
constant and $A$ is the amplitude of the motion. In this case, that quantity is: . $5(800 \mathrm{nts} / \mathrm{m})(.32$ $m)^{2}=40.96$ joules. As this is approximately 40 joules, this response is true.]
b.) The work done per stroke will be approximately 40 joules. [This is tricky in its simplicity. The work done, to a good approximation, will equal the energy available in the system (true, some of the work might be done overcoming friction, but it will still be done). That means that the answer to Response a and the answer to this response should be the same, and this response is true.]
c.) The power provided per stroke will be approximately 200 watts. [Power is defined as the work per unit time. If one stroke does 40 joules of work, and if one stroke takes $1 / 5$ of a second (remember, the frequency is 5 cycles/second, so the period is $1 / 5$ second per cycle), then the number of joules per second being provided to the system will be equal to the product (40 joules/cycle)/( $1 / 5$ second/cycle) $=200$ watts. This response is true.]
d.) All of the above. [Yup.]
e.) None of the above. [Nope.]
4.) A 12 kg projectile is fired straight up. Halfway to the top of its flight, its kinetic energy is 300 joules. Taking g's magnitude to be $10 \mathrm{~m} / \mathrm{s}^{2}$ :
a.) The projectile's velocity at the halfway point is $50 \mathrm{~m} / \mathrm{s}$. [The kinetic energy is $(1 / 2) \mathrm{mv}^{2}=.5(12 \mathrm{~kg}) \mathrm{v}^{2}=300$ joules. The math yields a velocity $\mathrm{v}=(50)^{1 / 2} \mathrm{~m} / \mathrm{s}$. This statement is false.]
b.) The projectile's maximum height is 5 meters. [At the halfway point, the total mechanical energy will be $U+K E=m g(h / 2)+300$. At the top, the total mechanical energy will be $\mathrm{U}+0=\mathrm{mgh}$. As the total energy will not change (no extraneous work being done during the motion), these two can be equated. Solving for h yields 5 meters. This response is true.]
c.) The projectile's initial velocity (i.e., at ground level) is $8 \mathrm{~m} / \mathrm{s}$. [The total mechanical energy is equal to the potential energy at the top of the flight. That quantity is mgh $=(12 \mathrm{~kg})\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)(5 \mathrm{~m})=600$ joules. The total energy is also equal to the kinetic energy at the beginning of the flight. That quantity is $(1 / 2) \mathrm{mv}^{2}=.5(12) \mathrm{v}^{2}$. Equating the two total energy expressions yields an initial velocity $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}$. This response is false.]
d.) None of the above. [Evidently not.]
5.) A graph of the velocity vs. position of a body oscillating in simple harmonic motion looks like:
a.)

b.)

C.)

d.)
e.) None of these.

[Commentary: If you will remember, the position function for this kind of motion is characterized by a sine wave whereas the velocity function is a cosine. Put a little differently, the position and velocity are out of phase with one another by a quarter of a cycle. What does this mean? Consider the sketch on the next page. Assume that when the body is at $x=-.5$ meters, its velocity is $+2 \mathrm{~m} / \mathrm{s}$. There will be a point in time when the bodyis at $\mathrm{x}=-.5$ meters
going the other direction at $-2 \mathrm{~m} / \mathrm{s}$. In other words, there will be two velocities associated with $x=-.5$ meters. How does one graph such a function? With a circle! In short, graph c does the trick.]
6.) During a 10 second period, a wheel has a constant torque applied to it that diminishes its angular speed from $1 \mathrm{rad} / \mathrm{sec}$ to $.8 \mathrm{rad} / \mathrm{sec}$. In 20 seconds, the angular speed will be:
a.) . $64 \mathrm{rad} / \mathrm{sec}$. [A constant torque will provide a constant angular acceleration. As such, we can use kinematics or, better yet, remember that a constant acceleration means that vel ocity is changing linearly with time. That being the case, doubling the time simply doubles the slowdown. The correct response should be $.6 \mathrm{rad} / \mathrm{sec}$, and this response is false.]
b.) $.60 \mathrm{rad} / \mathrm{sec}$. [This is the one.]
c.) $.40 \mathrm{rad} / \mathrm{sec}$. [N ope.]
d.) . $10 \mathrm{rad} / \mathrm{sec}$. [Nope.]
7.) At a given instant, a mass $m$ is observed to be moving with velocity $v_{o}$ up a frictional incline plane of angle $\theta$.
a.) The direction of the mass's acceleration is opposite the direction of its velocity. [The two accelerating forces, a gravitational component and friction, are oriented down the incline. As the velocity is up the incline, this statement is true.]
b.) The acceleration would be greater if the block were traveling down the incline. [If the body were traveling down the incline, the gravitational component would still be down the incline but the frictional force would be up the incline. As the accelerating forces would subtract from one another, the acceleration would be less than if the body was moving up the incline. This statement is false.]
c.) If $\theta$ were made smaller, the coefficient of kinetic friction would change, and the acceleration would be lessened. [The coefficient of friction is a constant. This statement is false.]
d.) Both a and b. [Nope.]
e.) Both a and c. [Nope.]
--The following information pertains to Problems 8 through 10: Projectiles $A$ and $B$ are fired at the same time from a height $h$ meters above the ground. Both projectiles have the same muzzle velocity $\mathrm{v}_{\mathrm{o}}$. It takes $\mathrm{t}_{1}$ seconds for Projectile A to get to the top of its flight. A

projectileA

projectile B sketch of the two situations is shown to the right.
8.) If Projectile A travels a net horizontal distance equal to $x_{1}$ meters:
a.) Projectile $B$ will travel a net horizontal distance less than $x_{1}$ meters. [If Projectile A is angled steeply--at almost $90^{\circ}$--it obviously won't go very far, and this statement will be false.]
b.) Projectile $B$ will travel a net horizontal distance equal to $x_{1}$ meters. [F rom the information given in Response a, this is clearly not true.]
c.) Projectile $B$ will travel a net horizontal distance greater than $x_{1}$ meters. [If Projectile $A$ is angled close to $45^{\circ}$, the projectile will have both an $x$ and $y$ component of velocity and, in fact, will land close to its maximum range--definitely farther than $\mathrm{x}_{1}$. This statement is clearly false.]
d.) The relative distances between Projectiles A and B will depend upon the angle $\theta$. [E verything said above makes it clear that the angle is crucial in determining the range.]
9.) The initial velocity of Projectile $B$ will:
a.) Be equal to the initial velocity of Projectile A. [Velocity is a vector. It has direction. The magnitudes of the two velocities are the same, but the directions are not. This statement is false.]
b.) Be equal to the velocity of Projectile $A$ at timet ${ }_{1}$. [At time $t_{1}$, Projectile $A$ will be at the top of its arc moving only in the horizontal. For that case, the direction of the two velocity vectors will be the same (both will be in the horizontal), but the magnitudes will be different (one is $\mathrm{v}_{\mathrm{o}^{\prime}}$ the other $\mathrm{v}_{0} \cos \theta$ ). This statement is false.]
c.) Be equal to the velocity of Projectile A just an instant before touch down. [This is obviously false.]
d.) None of the above. [This is it.]
10.) Consider the graphs shown below:

a.)


c.)

d.)





i.)

j.)

Projectile B's:
a.) Y-component of its Position vs. Time graph looks like graph a.
b.) X-component of its Position vs. Time graph looks like graph j .
c.) Magnitude of its Velocity vs. Time graph looks like graph g.
d.) X-component of its Velocity vs. Time graph looks like graph c.
e.) Y-component of its Acceleration vs. Time graph looks like graph b.
[Commentary: A graph for each of the major parameters for this situation is shown below. This is something you should have been able to both visualize and sketch on your own. If you think you wouldn't have been able to do that, use the graphs provided as a stimulus to do the visualization part.]

-----------end of
section
11.) A car moving with velocity $v_{o}$ rounds a flat curve of radius $R$. If the car had been moving a bit faster, it would have lost traction and skidded.
a.) If the car had broken traction and skidded, it would have moved away from the curved path it was supposed to take. The force that moved it out of its curved path is called a centrifugal force. [This is twisted. As any driver who has every hit an ice patch knows, a car that loses traction while rounding a curve will simply move in a force free straight line (the apparent outward movement of the car is an illusion brought on by the fact that the road is curved). The idea of a centrifugal force is misleading. Centrifugal force, in fact, does not exist (though centrifugal effects do exist) except as a fictitious force that must be assumed to exist if one is to use N.S.L. while examining a problem from the perspective of an accelerated frame of reference (like the frame that is attached to you as you sit behind the wheel of a car moving in a curved path). Summarily, this statement is false.]
b.) Kinetic friction in this situation will be less than $\mathrm{mv}_{0}{ }^{2} / R$. [This is a tricky question. Because the set-up has nothing to do with kinetic friction (it is static friction that keeps a car from skidding sideways when rounding a turn), the temptation is to assume that we are looking at apples and oranges and that the statement is false. What is important to notice is the fact that there still is a relative relationship between static and kinetic friction in the sense that the kinetic friction between two surfaces is always less than the static friction between those same two surfaces. As the static frictional force must equal $\mathrm{mv}_{0}{ }^{2} / \mathrm{R}$ in this case, the kinetic frictional force must be less than that amount, and this statement is true.]
c.) The coefficient of static friction is equal to $\mathrm{mv}_{0}{ }^{2} / \mathrm{R}$. [The coefficient of static friction is a unitless constant. If for no other reason, this statement can't be true because $\mathrm{mv}_{0}^{2} / \mathrm{R}$ is not a unitless quantity. In fact, this is supposed to be a trick question. If you had read static friction instead of coefficient of static friction, the statement would have been true. That is, Newton's Second Law states that the net force acting on a body must equal the product of two variables--the body's mass and the body's acceleration. In the case of a car rounding an unbanked curve, the center-seeking force that must exist naturally within the system to pull the car out of straight-line motion and into curved motion is provided solely by the static friction (or a component of the static friction) between the car and the road. We don't know anything about static friction in this problem, but because it is the only force acting it must equal $m a=m\left(v^{2} / \mathbb{R}\right)$. If the static frictional force is equal to this value, the car will be right at its limit and will not slide. This response is false.]
d.) None of the above. [Nope.]
12.) A 10 kg projectile moving vertically has 1500 joules of gravitational potential energy at the same time that it has 1500 joules of kinetic energy. (Assume the gravitational potential energy function is defined as zero at ground level, and assume g's magnitude is $10 \mathrm{~m} / \mathrm{s}^{2}$ ). At this point:
a.) The body's velocity is exactly half that of its maximum value. [With the total energy equal to 3000 joules, we can write $3000=.5 \mathrm{mv}_{\max }{ }^{2}=.5(10) \mathrm{v}_{\max }{ }^{2}$. Solving yields a maximum velocity of $(600)^{1 / 2}=24.5 \mathrm{~m} / \mathrm{s}$. The velocity at the point we are dealing with can be determined using $1500=.5 \mathrm{mv}_{\max }^{2}=.5(10) \mathrm{v}_{1}^{2}$. That velocity is $(300)^{1 / 2}=17.3 \mathrm{~m} / \mathrm{s}$. This statement is false.]
b.) The body's vel ocity is more than half that of its maximum value. [F rom Response a, this is true. This might be obvious, depending upon whether you are good at looking at situations in a qualitative way. The body is at its kinetic energy halfway point. If kinetic energy were a linear function of vel ocity, the body's velocity at the halfway point would be $\mathrm{v}_{\max } / 2$. Because kinetic energy is related to the velocity squared, the body in this case would have a velocity of $v_{\max } / 2^{1 / 2}$.]
c.) The body's velocity is less than half of its maximum value. [From above, this is false.]
d.) This cannot be answered as the maximum velocity is not a quantity we can calculate. [Nope.]

13.) A string is attached to the ceiling at one end and has its other end wrapped around an axle of mass $m$ and radius $R$ (see sketch). Attached to the axle are two wheels of mass $m$ and radius $3 R$ each, one at each end. The moment of inertia of the entire assembly along the axle's length is $1.5 \mathrm{mR}^{2}$. All numerical values are measured in the MKS system. The system is released to freefall. During the fall, the acceleration of the body's center of mass is:
a.) . 40 g . [Summing the torques about the center of mass yields: $-\mathrm{TR}=-\mathrm{I} \mathrm{cm} \alpha$. Using a $=R \alpha$ and $\mathrm{I}_{\mathrm{cm}}=1.5 \mathrm{mR}^{2}$, and multiplying by -1 , we get $T=\mathrm{I}_{\mathrm{cm}} \mathrm{a} / \mathrm{R}=\left(1.5 \mathrm{mR}^{2}\right)(\mathrm{a} / \mathrm{R}) / \mathrm{R}=$ 1.5ma. Summing the forces in the vertical yields: $\mathrm{T}-(3 \mathrm{~m}) \mathrm{g}=-(3 \mathrm{~m}) \mathrm{a}$. Substituting in for T yields: $1.5 \mathrm{ma}-3 \mathrm{mg}=-3 \mathrm{ma}$, or $\mathrm{a}=.67 \mathrm{~g}$. If you got the negative sign right on the translational acceleration part, but inadvertently used minstead of 3m for your mass, you undoubtedly got .4 g for your solution. Unfortunately, that is wrong and this response is false.]
b.) .67 g . [This is the one.]
c.) 2.0 g . [ Nope .]
d.) None of the above. [Nope.]
14.) A 1 kg mass is found to be moving $18 \mathrm{~m} / \mathrm{s}$ up a $30^{\circ}$ incline. How fast is the mass moving 3 seconds later? Take g to be $10 \mathrm{~m} / \mathrm{s}^{2}$.
a.) $2 \mathrm{~m} / \mathrm{s}$. [The modified conservation of momentum equation can be written as $\Sigma p_{1}+\Sigma \mathrm{F}_{\text {ext }} \Delta \mathrm{t}=\Sigma \mathrm{p}_{2}$. What it says is that in a particular direction, the momentum of a system will stay the same over time $\Delta t$ unless there are external impulses (the $F_{\text {ext }} \Delta t$ quantities) acting on the system during that time. If external impulses are present, the total momentum at time $t_{2}$ will equal the total momentum at time $t_{1}$ plus or minus the extra impulses that are applied. Taking the line of the incline to the $x$ direction, we can use the modified conservation of momentum equation to write: $\Sigma \mathrm{p}_{1}+\Sigma \mathrm{F}_{\mathrm{ext}} \Delta \mathrm{t}=\Sigma \mathrm{p}_{2}$, or $(1 \mathrm{~kg})(18 \mathrm{~m} / \mathrm{s})+\left[-(1 \mathrm{~kg})\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)(\mathrm{sin}\right.$ $\left.\left.30^{\circ}\right)\right](3 \mathrm{sec})=(1 \mathrm{~kg}) \mathrm{v}_{2}$, or $\mathrm{v}_{2}=3 \mathrm{~m} / \mathrm{s}$. If this be the case, this response is false.]
b.) $3 \mathrm{~m} / \mathrm{s}$. [F rom the above analysis, this is a true statement.]
c.) $6 \mathrm{~m} / \mathrm{s}$. [Nope.]
d.) None of the above. [Nope.]
15.) A satellite of mass $m$ orbits a planet of mass $M$ and radius $R$ in a circular orbit. If the orbit is $3 R$ units above the planet's surface, the velocity of the satellite must be:
a.) $[G M /(3 R)]^{1 / 2}$. [Looking for a velocity in an ORBIT problem usually means that you will have to use N.S.L. coupled with the fact that the orbiting body's acceleration is centripetal. As the distance between the center of masses of the two objects is 4R, N.S.L. yields: $G m M /(4 R)^{2}=m\left[v^{2} /(4 R)\right]$, or $v=[G M /(4 R)]^{1 / 2}$. If you inadvertently took the distance part of the force expression to be 3R (i.e., the distance between the satellite and the surface of the planet), you got the incorrect response . . . which is to say the one offered in Response a.]
b.) $[G M /(4 R)]^{1 / 2}$. [F rom the discussion above, this is the correct response.]
c.) $\left[G M /\left(16 R^{2}\right)\right]^{1 / 2}$. [Nope.]
d.) $\mathrm{GM} /(4 \mathrm{R})$. [ Nope .]
16.) During time $\mathrm{t}_{1}$ a single, constant force is applied to body $A$ changing its velocity from 0 $\mathrm{m} / \mathrm{s}$ to $30 \mathrm{~m} / \mathrm{s}$. For an unknown period of time $t_{2}$ the same force is applied to a second, identical body (call it B). During $\mathrm{t}_{2}$, B's velocity changes from zero to $15 \mathrm{~m} / \mathrm{s}$.
a.) The net work done on A will be twice as much as done on B . Also, the distance over which the force is applied to $A$ is twice that applied to $B$. [F or the first part, we know that $W_{\text {net }}=$ $K E_{2}-K E_{1}$. As both bodies start from rest, the question is whether $(1 / 2) \mathrm{mv}_{2, \mathrm{~A}}{ }^{2}=2(1 / 2) \mathrm{mv}_{2, B}{ }^{2}$ ? Clearly, $\mathrm{v}_{1, \mathrm{~A}}{ }^{2}=(30 \mathrm{~m} / \mathrm{s})^{2}=900 \mathrm{~m}^{2} / \mathrm{s}^{2}$ is not the same as twice $\mathrm{v}_{2, \mathrm{~B}}{ }^{2}=(15 \mathrm{~m} / \mathrm{s})^{2}=225 \mathrm{~m}^{2} / \mathrm{s}^{2}$. This answer is false without even having to look at the second part.]
b.) The net work done on A will be twice as much as done on B, but the distance over which the force is applied will not be twice as long. [This is false for the same reason Response a was wrong.]
c.) The net work done on A will be less than twice as much as done on B. Also, the distance over which the force is applied to $A$ is less than twice that for $B$. [The first part of this is clearly wrong. This statement if false.]
d.) The net work done on A will be more than twice as much as done on B. Also, the distance over which the force is applied to $A$ is more than twice that for $B$. [As shown in Response a, the first half of this statement is true. As for the second part: We know that $\mathrm{W}=$ F.d. Put another way, the amount of work a constant force does is proportional to the distance over which it is applied. If the work done to A is more than twice as much as is done to B , then the distance over which the force is applied must also be more than twice for A than for B . This statement is true.]
e.) None of the above. [Nope.]
17.) A pendulum bob's mass is decreased by a factor of 4 while its length is increased by a factor of 4.
a.) Its frequency will stay the same as will its period. [The angular frequency of a pendulum is equal to $(\mathrm{g} / \mathrm{L})^{1 / 2}$. This means that changing the mass of the bob does nothing to the angular frequency, the frequency, or the period, but changing the length $L$ does change things. In fact, increasing $L$ decreases the angular frequency and, by extension, the frequency. As the period is the inverse of the frequency, decreasing the frequency increases the period. On every level, this response is false.]
b.) Its frequency will increase and its period will increase. [From what has been said above, this response is false.]
c.) Its frequency will decrease and its period will increase. [From what has been said above, this is the one.]
d.) Its frequency will decrease and its period stays the same. [Nope.]
e.) Its frequency will increase and its period will decrease. [Nope.]
18.) For small amplitude oscillations, the force on an unusually designed spring is very close to being proportional to the displacement of the spring. For large amplitude oscillations, the force becomes non-linear. A graph of the force vs. displacement is shown to the right. When this spring executes large amplitude oscillations:
a.) Its period will be the same as in the small oscillations case, and its
 frequency will also be the same. [Period and frequency are both qualitatively related to the vigorousness of the restoring force. As the restoring force is not linear in this case for larger amplitude oscillations, this response must be false.]
b.) Its period will be smaller than in the small oscillations case, and the frequency will also be smaller. [The fact that the period and frequency are both getting smaller tells you that this response is false. Frequency and period are inversely related.]
c.) Its period will be smaller than in the small oscillations case, but the frequency will be larger. [If the force had been linear with position, it would have taken a certain amount of time for one cycle. If at any time during the motion the force is larger than would have been the case for the linear situation, the body will accelerate more quickly back toward equilibrium and the amount of time for one cycle (i.e., the period) will be less than for the linear situation. With a smaller period, we get a larger frequency, and this response is true.]
d.) Its period will be larger than in the small oscillations case, but the frequency will be smaller. [Nope.]
e.) None of the above. [Nope.]
19.) A string carries a transverse wave down its axis. A slinky carries a longitudinal wave down its axis. At a given instance, the string looks like the top sketches on each grid below. Assuming the slinky's frequency is twice that of the wave on the string, which of the slinky waveform matches this situation?

[Commentary: The frequency tells you how many cycles pass you by per unit time. That means that if the string's frequency is 1 cycle/second, the slinky's frequency will be 2 cycles/second. If the wave vel ocity for the two waveforms was the same, that would mean that the string's wavelength would be twice that of the slinky (wavelength and frequency are inversely related) and graph d would have been the correct sol ution. Unfortunately, we don't know that the wave velocities are the same. In fact, they most probably will not be the same. As such, the only response we can give is Response e.]
20.) Three frictionless inclines are shown below. The sketches are drawn approximately to scale with the curved incline being a quarter circle. The masses are different sizes. All three masses are released at once. Mass $m_{2}$ travels the same distance as does mass $m_{3}$.


The first mass to the bottom will be:
a.) Mass $m_{1}$. [The mass that gets to the bottom first will be the mass with the largest
acceleration and the shortest distance to travel. This isn't the one.]
b.) Mass $m_{2}$. [Although masses $m_{2}$ and $m_{3}$ travel the same distance, $m_{3}$ 's acceleration, on average, is greater than $m_{2}$ 's acceleration. This isn't the one.]
c.) Mass $m_{3}$. [From what has been said above, this is the true statement.]
d.) Mass $m_{2}$ and $m_{3}$. [Nope.]
e.) They will all reach the bottom at the same time. [Nope.]
21.) A metal hoop and a hollow rubber ball have the same radius and mass. The two are placed side by side at the top of an incline.
a.) The hoop will arrive at the bottom first, but its rotational kinetic energy will be less than the rotational kinetic energy of the ball when it arrives at the bottom. [The body with the greatest moment of inertia (i.e., the body that most resists changes of its angular motion) will take the most time to get to the bottom. As the hoop's mass is distributed as far from the axis of rotation as possible, the hoop has the greatest rotational inertia. As such, it will not reach the bottom first. This response is false.]
b.) The hoop will arrive at the bottom first, and its rotational kinetic energy will be greater than the rotational kinetic energy of the ball when it arrives at the bottom. [Again, this is false because the hoop doesn't arrive first at the bottom.]
c.) The ball will arrive at the bottom first, but its rotational kinetic energy will be less than the rotational kinetic energy of the hoop when it arrives at the bottom. [From what has been stated above, the first part of this is correct--the ball does reach the bottom first. As for the rotational kinetic energy part, let's think through the problem: Both objects begin with the same amount of potential energy (same masses, same height above the incline's bottom). As each object begins to roll, energy is redistributed between translational kinetic energy (.5mv ${ }^{2}$ ) and rotational kinetic energy (.5I $\omega^{2}$ ). The body that makes it to the bottom first will have the
larger translational velocity, so it will have the larger translational kinetic energy. As the translational and rotational kinetic energies must always add to the same amount (energy is conserved), the ball with the greatest translational kinetic energy must have a smaller rotational kinetic energy. In short, this response is true.]
d.) The ball will arrive at the bottom first, and its angular velocity will be greater than the angular velocity of the hoop when it arrives at the bottom. [From the analysis above, this response is false.]
22.) A block of mass 2 kg is found sliding with velocity $5 \mathrm{~m} / \mathrm{s}$ against a frictional, circular wall of radius 3 meters. If the frictional force is a constant 4 newtons:
a.) The net force acting on the block at the instant alluded to is 4 newtons, and the distance the block travels before coming to rest is 1.25 meters. [The net force acting on the block must include the center seeking force that is motivating the block into circular motion. As 4 newtons is only the frictional component of the net force, this statement is false.]
b.) The net force acting on the block at the instant alluded to is 16.7 newtons, and the distance the block travels before coming to rest is 1.25 meters. [One approach in determining the net centripetal force is to sum the forces acting in the center seeking direction. Unfortunately, that sole force is the normal force between the block and the wall. If we knew the coefficient of friction between the block and wall, we could use our knowledge of the frictional force to determine N. Unfortunately, we haven't that information. The only other alternative is to determine a number that is equal to the net center seeking force. We can do that because we know from N.S.L. that the sum of the forces will equal ma, where $a=v^{2} / R$ in this case. Putting in the numbers, we get $\mathrm{mv}^{2} / \mathrm{R}=(2 \mathrm{~kg})(5 \mathrm{~m} / \mathrm{s})^{2} /(3$ meters $)=16.7$ newtons. Unfortunately, that is only the centripetal component to the net force (we have forgotten to include friction). This statement is false.]
c.) The net force acting on the block at the instant alluded to is 17.1 newtons, and the distance the block travels before coming to rest is 6.25 meters. [ From above, we know that $\mathrm{F}_{\mathrm{C}}=$ 16.7 nt and $\mathrm{F}_{\text {tangential }}=4$ newtons. As these are at right angles to one another, the magnitude of the two added vectorially together yields $F=\left(F_{c}{ }^{2}+F_{\text {tangential }}{ }^{2}\right)^{1 / 2}=\left[(16.7 n t)^{2}+(4 n t)^{2}\right]^{1 / 2}=17.1$ newtons. Great jumping huzzahs, this is the answer to the first part. As far as the second part goes, conservation of energy maintains that $.5 \mathrm{mv}^{2}-\mathrm{fd}=0$, or $\mathrm{d}=.5 \mathrm{mv}^{2} / \mathrm{f}=.5(2 \mathrm{~kg})(5 \mathrm{~m} / \mathrm{s})^{2} /(4$ $\mathrm{nts})=6.25$ meters. This statement is true.]
d.) None of the above. [Nope.]
23.) A graph of a traveling wave as seen at $t=2$ seconds and $t=3$ seconds is shown to the right. The wave vel ocity of the wave is:
a.) $18 \mathrm{~m} / \mathrm{s}$. [Between $\mathrm{t}=2$ seconds and $\mathrm{t}=3$ seconds, the wave traveled 18 meters. As such, the wave velocity is $18 \mathrm{~m} / \mathrm{s}$, and this response is true.]
b.) $6 \mathrm{~m} / \mathrm{s}$. [ Not this one.]
c.) $4 \mathrm{~m} / \mathrm{s}$. [ Not this one.]
d.) None of the above. [Not this one.]

24.) A ladder of mass $m$ and length $L$ sits on a frictionless floor perched against a frictionless wall. A force $F$ acting at a distance $d$ units up the ladder keeps the ladder from angularly accelerating. If $F$ is removed:
a.) There will be only two forces acting on the ladder as it begins to accelerate. [Nope, there are three-the vertical force at the floor, gravity, and a normal force at the wall. This response is false.]
b.) The vertical force V acting at the floor will equal mg . [N.S.L. yields $V-m g=m a y$. If the system was not accelerating, the vertical force acting at the floor (i.e., the only force acting at the floor) will, indeed, equal
 mg . As $\mathrm{a}_{\mathrm{y}}$ is not zero, this statement is false.]
c.) The normal force at the wall must go to zero. [Again, if the body was not accelerating, $a_{x}$ would be zero, and $N$, the only force acting in the horizontal, would have to be zero. Unfortunately, the body is accelerating, so $N$.S.L. yields $N=m a_{x}$, and $N$ is not equal to zero. This response is false.]
d.) None of the above. [From what has been said above, this is it.]
25.) A spinning skater has rotational kinetic energy equal to 32 joules when spinning with an angular velocity of $4 \mathrm{rad} / \mathrm{sec}$. If the skater had kept the same position but had spun with an angular velocity of only $2 \mathrm{rad} / \mathrm{sec}$, his kinetic energy would have been:
a.) Half the original amount, or 16 joules. [The question is, how does halving the angular velocity affect kinetic energy, assuming that all else is kept the same? As kinetic energy is a function of angular velocity squared, halving the angular vel ocity should quarter the kinetic energy. As such, the answer is 8 joules, and this response is false.]
b.) A quarter the original amount, or 8 joules. [This is the one.]
c.) Double the original amount, or 64 joules. [Nope.]
d.) Quadruple the original amount, or 128 joules. [Nope.]
26.) A box of mass $m$ sits on the bed of a flatbed pick-up truck. The maximum acceleration the truck can experience without the box breaking loose is $a_{1}$. Assume the truck accelerates to the right.
a.) The coefficient of static friction must be equal to $a_{1} / g$. [To determine
 this, we must do an f.b.d. on the box (shown to the right), then sum the forces. To begin with, note that the STATIC FRICTIONAL FORCE is to the right. (There are two ways to see this. First, from a fixed frame of reference (i.e., the ground) the box must move to the right. It might not move to the right as fast as the truck does (hence appearing to move to the left relative to the truck's bed), but it will move to the right somewhat. The only force available to effect that acceleration is friction. The other way to look at it is to consider the direction the block would slide relative to the truck bed if the box broke loose and began to slide. Relative to the bed, it would slide leftward. The static frictional force must be opposite that direction, or to the right.) N.S.L. yields $\mu_{\mathrm{k}} \mathrm{mg}=\mathrm{ma}_{1}$, or $\mu_{\mathrm{k}}=\mathrm{a}_{1} / \mathrm{g}$. This statement is true, but are there others?]
b.) The direction of the static frictional force acting on the box is to the left. [According to the arguments used above, this is false.]
c.) The direction of the static frictional force acting on the box is to the right. [According to the arguments presented above, this statement is true.]
d.) Both a and b. [Not so.]
e.) Both a and c. [This is the one.]
27.) A position versus time graph for a body oscillating in simple harmonic motion is shown to the right. The motion's angular frequency is:
a.) $\pi / 12$ radians/second. [The time span between any two consecutive peaks or troughs is 12 seconds, so the period of the oscillation is 12 seconds per cycle. The inverse of that is the frequency at ( $1 / 12$ ) cycle/second, and the angular frequency is that value multiplied by $2 \pi$, or $\pi / 6$ radians per second. As such, this response is
 false.]
b.) $\pi / 6$ radians/second. [From above, this is the one.]
c.) 1/12 radians/second. [Nope.]
d.) None of the above. [Nope.]
28.) When a 10 kg body is .5 meters up a $30^{\circ}$ incline plane, its velocity up the incline is observed to be $20 \mathrm{~m} / \mathrm{s}$. A constant 50 newton frictional force acts on the body.
a.) The net work done on the body will be negative and the body will slow down. ©Note: This is an example of a problem that starts out looking like a beast only to be found to be a pussy cat. It is also an example of a typical A.P. maneuver--putting an incredibly EASY question in the middle of the test (versus putting the easy stuff at the beginning and having things get harder as you go). The moral of the story: When taking the A.P. test, don't assume that the last few questions will be hard--they may be gimmees.\} [With the body moving up the incline, it is obviously slowing down. That means that energy is being removed from the system by negative work. This answer is true.]
b.) The net work done on the body will be positive and the body will slow down. [J ust because the body is moving in the positive direction doesn't mean positive work is being done on it. This statement is false.]
c.) The body will have come to rest at the end of the 4 meters of motion. [H opefully you have deduced that Response a is true. If Response d had been "Both a and c," then you'd have to determine whether this statement was true along with Response a. As Response dincludes a choice you know is not true, and as there can only be one true response per question, this has to be false. I will show that this is true by doing the math ONLY because it might be educational for you, but if this was a real A.P. test you were taking, you'd be dumb to do the same. Time is of the essence on A.P. tests. Be clever--don't do any more than you absolutely have to do. The solution: The modified conservation energy equations is $\mathrm{KE}_{1}+\mathrm{U}_{1}+\mathrm{F}_{\text {extraneous }} \cdot \mathrm{d}=\mathrm{KE}_{2}+\mathrm{U}_{2}$. Assuming that $d$ is the distance up the incline (that means that $d \sin \theta$ is the change of vertical position) to the stop point and the zero potential energy level is defined as the position of the body when its velocity is $20 \mathrm{~m} / \mathrm{s}$ (i.e., .5 meters up the incline), and remembering that (1.) the mass is 10 kg , (2.) the initial velocity is $20 \mathrm{~m} / \mathrm{s}$, and (3.) friction does -fd's worth of work as the body moves to its resting point, we can write:

$$
\begin{gathered}
.5 \mathrm{mv}_{1}{ }^{2}+0+(-\mathrm{fd})=0+\mathrm{mg}(\mathrm{~d} \sin \theta) \\
.5(10 \mathrm{~kg})(20 \mathrm{~m} / \mathrm{s})^{2}-(50 \mathrm{nt}) \mathrm{d}=(10 \mathrm{~kg})\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\mathrm{d} \sin 30^{\circ}\right) \\
\Rightarrow \quad \mathrm{d}=20 \text { meters. }
\end{gathered}
$$

This statement is false.]
d.) Both band c. [Nope.]
29.) A 2 kg mass with 100 joules of kinetic energy strikes a second body that is half the mass of the first. The collision is perfectly inelastic. What is the approximate vel ocity of the two bodies after the collision?
a.) $10.0 \mathrm{~m} / \mathrm{s}$. [Noting that $.5 \mathrm{mv}^{2}=.5(2 \mathrm{~kg}) \mathrm{v}^{2}=100$ joules implies that $\mathrm{v}=10 \mathrm{~m} / \mathrm{s}$, this response is actually the initial velocity of the 2 kg mass. As this would not be the after-collision velocity as well, this is false.]
b.) $8.16 \mathrm{~m} / \mathrm{s}$. [If energy were conserved through the collision, you could write 100 joules $=.5(2 \mathrm{~kg}+1 \mathrm{~kg}) \mathrm{v}_{\text {after }}{ }^{2}$, or $\mathrm{v}_{\text {after }}=8.16 \mathrm{~m} / \mathrm{s}$. Unfortunately, energy is not conserved, and this response is false.]
c.) $6.7 \mathrm{~m} / \mathrm{s}$. [Conservation of momentum coupled with the initial velocity found in Response a (i.e., $10 \mathrm{~m} / \mathrm{s}$ ) yields $(2 \mathrm{~kg})(10 \mathrm{~m} / \mathrm{s})=(3 \mathrm{~kg}) \mathrm{v}_{\text {after }}$, or $\mathrm{v}_{\text {after }}=6.7 \mathrm{~m} / \mathrm{s}$. This response is true.]
d.) $3.3 \mathrm{~m} / \mathrm{s}$. [Nope.]
30.) Two asteroids, both of mass $M$ and radius $R$, orbit one another in circular paths (the origin of the coordinate axis is at the center of mass of the two asteroids). A mass $m$ is moved
 from the left asteroid to the right asteroid. The distance between the two asteroids is 10 R (see sketch). Which of the graphs shown describe the force on the mass and the mass's potential energy at various points as it makes its trip?
a.)

b.)

c.)

d.)
e.) None of these.
at 5 R relative to c . of m .
[Commentary: Think about the gravitational force acting on the mass when it is at -5R (i.e., when it is on the surface of the left-hand asteroid). The predominate pull at that point will be from the left-hand asteroid in the negative direction, relative to the coordinate axis centered at the system's center of mass (taking into account the considerably smaller force in the positive direction exerted by the right-hand asteroid, the net force on the mass is: $-\mathrm{GMm} /(5 \mathrm{R})^{2}+$ $\left.\mathrm{GMm} /(10 \mathrm{R})^{2}\right)$. As the mass proceeds toward the center of the system, the force will diminish until it reaches zero right at the center of mass of the system. How will the force diminish? It will drop by $1 / r^{2}$ (think about the force expression). This eliminates Response a and c (Response $c$ is eliminated because that force function does not approach the origin as $1 / r^{2}$ ). Once the mass has passed the center of the system, the direction of the net force changes to positive and gets larger as the body gets closer to the right-hand asteroid. That part of the graph will be an inverted mirror image of the force function as it exists on the left-hand side of the system's center (that means Response $b$ and $d$ are still viable). Looking at the potential energy function, Response $d$ can be immediately eliminated if we remember that the gravitational potential energy function associated with far-distant objects is always negative (i.e., it's - GmM/r). The graph shown in

Response $b$ is a $-1 / r$ function, but one might question whether it is acceptable. Why might there be a problem? Because the object clearly has no net gravitational force acting on it at the center (the sum of the asteroid forces at that point will equal zero), but the graph suggests that the mass nevertheless has potential energy there. This is where a simplistic understanding of potential energy functions can get you into trouble. If you believe that when a body has potential energy it will always accelerate when released (common example: hold a pencil above the floor and let it go--it has potential energy and, when released, it accelerates downward), you must conclude that Response b cannot be true. Why? Because there is no net gravitational force at that point, yet the graph maintains that there is gravitational potential energy. The problem here can be resolved if we remember that, by definition, the gravitational potential energy associated with each object in the celestial system is defined as zero at infinity. With that being the case, the net potential energy due to the presence of an assemblage of large bodies turns out to be - $G m_{1} \mathrm{~m} / \mathrm{r}_{1}-G m_{2} \mathrm{~m} / \mathrm{r}_{2}$ - $\mathrm{Gm}_{3} \mathrm{~m} / \mathrm{r}_{3} \ldots$. etc. In short, the net potential energy held by the mass when at system's center will not be zero, it will be $-G m_{p} m_{0} /(6 R)-G m_{p} m_{0} /(6 R)=-2 G m_{p} m_{0} /(6 R)$. Is this kosher? Sure it is! As long as the difference in the evaluation of the potential energy between any two points tells you how much work the net gravitational field does as a body moves between the two points, that's all that is important. The fact that the body has potential energy at some point but will not accelerate if released at that point is inconsequential. In short, Response b is OK.]
31.) The position function for an oscillating body is $x=20 \sin (.6 t-\pi / 2)$. At $t=0$, the body is at:
a.) Equilibrium. [You can do this in one of two ways. You can either plug $t=0$ into the position expression and see what you come up with, or you can note that at $\mathrm{t}=0$, the position of the body is governed by the phase shift, which for this problem is $-\pi / 2$. This phase shift pushes the axis to the left a quarter of a cycle (see sketch), which means the $x$ position will be negative and as big as it can be. In short, it will not be at equilibrium. This is false.]
b.) Its maximum positive displacement. [Nope.]
c.) Its maximum negative displacement. [Yup.]

d.) N one of the above. [Nope.]
32.) The moment of inertia about the central axis of a disk (call it disk $A$ ) of radius $R$ and mass $m$ is known to equal $I_{A}$. A similar disk (call this disk $B$ )has half the mass of $A$ and half its radius.
a.) The moment of inertia about disk B's central axis will be the same as that of disk A. [Even if you don't remember the actual expression for the moment of inertia of a disk (it happens to be ( $1 / 2$ ) $\mathrm{mR}^{2}$ ), all moment of inertia expressions have mass and a distance squared term multiplied together (i.e., an $\mathrm{mR}^{2}$-type quantity). Completely ignoring the constant that goes out in front of that expression, we can deduce that halving the radius and mass will take on a mathematical form that is something like $(m / 2)(R / 2)^{2}=(1 / 8) m R^{2}$. In short, the moment of inertia will drop to $1 / 8$ of disk A's moment of inertia. This response is false.]
b.) The moment of inertia about disk B's central axis will be (1/2)I $A$. [From the discussion above, this response is false.]
c.) The moment of inertia about disk B's central axis will be (1/4) ${ }_{A}$. [F rom the discussion above, this response is false.]
d.) None of the above. [This is the one.]
33.) Two blocks on a frictionless surface are attached by a string (see sketch). If a 14 newton force is applied as shown, the tension in the string will be:

a.) 2.0 nts. [An f.b.d. on the two masses is shown to the right.

Using it, the acceleration of the system is found to be $2 \mathrm{~m} / \mathrm{s}^{2}$. From $\mathrm{m}_{5}$ 's f.b.d., $\mathrm{T}=5 \mathrm{a}=10 \mathrm{nts}$, and this statement is false.]
b.) 2.8 nts . [The statement is false.]
c.) 4.67 nts . [The statement is false.]
d.) 10.0 nts. [The statement is true.]

34.) A stationary mass A is struck head on elastically by a second body $B$ of the same mass and whose velocity before the collision is $\mathrm{v}_{\mathrm{o}}$. After the collision, mass A 's velocity is:
a.) Zero. [When mass B strikes an equal sized stationary mass A, B will stop dead and A will leave with B's initial velocity (this is all a consequence of the conservation of both energy and momentum for the situation). That means mass B's final velocity must be zero, while mass A's final velocity is $\mathrm{v}_{\mathrm{o}}$. This response is false.]
b.) $v_{0} / 4$. [F rom above, this won't do.]
c.) $v_{0} / 2$. [From above, this won't do.]
d.) $\mathrm{v}_{\mathrm{o}}$. [From above, this is the one.]
e.) None of the above. [From above, this won't do.]
35.) A 1 kg body sits a distance $\mathrm{h}=2$ meters above a tabletop. The tabletop is $\mathrm{d}=1.2$ meters high. Three students calculate the body's potential energy. Assuming the magnitude of the gravitational acceleration is $10 \mathrm{~m} / \mathrm{s}^{2}$, which student got it right?
a.) Student A with a calculated potential energy of 20 joules. [If you take the tabletop to be the zero potential energy level, $\mathrm{mgh}=(1 \mathrm{~kg})\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)(2 \mathrm{~m})=20$ meters. Although this statement is true, there is a problem. There may be other acceptable responses.]
b.) Student B with a calculated potential energy of 32 joules. [If you take the ground to be the zero potential energy level, $\mathrm{mgh}=(1 \mathrm{~kg})\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)(3.2 \mathrm{~m})=32$ meters. Although this statement is true, there is a problem. There may be other acceptable responses.]
c.) Student C with a calculated potential energy of 0 joules. [If you take the two meters above the tabletop to be the zero potential energy level, the potential energy of the body at that point will be zero. There is nothing wrong with this. Potential energy functions are usually defined as zero where the force is zero, but there is no place near the surface of the earth where gravity is zero. As such, this constant force's potential energy function can be defined to be zero wherever it's convenient. This statement, along with all of the others, is true.]
d.) Any of the three could be correct depending upon where the zero level for the potential energy function was defined. [For the reasons outlined above, this is it.]

